Fuzzy Graph Modeling and its Analysis System

Akira SATOH [1], Hajime YAMASHITA[2], Hiroshi SUDA[3],
Kensei TSUCHIDA [1] and Mitsuhiro NAMEKAWA[4]

1)Toyo Univ. 2)Waseda Univ. 3)Keio Univ. 4)Kaetsu College.

ABSTRACT Inexact information is used in expressing or describing human behaviors and mental processes. The information produced depends upon a person's subjectivity and is difficult to process objectively. The fuzzy graph model will make it possible to quantitatively process fuzzy information. Fuzzy information can be analyzed by using a fuzzy graph. In order to clarify the main feature of a fuzzy graph, we would represent it as an approximate graph and extract characteristics of a fuzzy graph, such as relational and similar structures. Thus, we are obliged to process many kinds of information concerning the structure of a fuzzy graph, and also draw quickly and display comprehensively fuzzy graphs and the related graphs. Fuzzy graphs having a few nodes are quickly draw, and easily and comprehensively analyzed. However, it is practically impossible to process a fuzzy graph having more than ten nodes by using the same method. Consequently, a new method for approximate analysis of fuzzy graphs is desired. To fill these needs, we propose a computer-aided system for analyzing fuzzy graphs through human interaction. This system can quickly and comprehensively draw a partition tree and a specified shape of approximate fuzzy graph. This paper describes the algorithm, methodologyand human interface of this system, and confirms effective analysis and evaluation of fuzzy phenomena. Example of potential application is sociometric analysis in psychology and cognition analysis in education.

1. INTRODUCTION

Inexact information are used in expression of human behaviors and mental processes. The information produced molded by a personal subjectivity. Thus it is difficult to measure objectively, analyze exactly and evaluate realistically. To date, it has not been possible to build up a model for a system including inexact information. But fuzzy theory can be applied as a effective strategy. Here it is desirable to objectively process inexact information by means of the fuzzy theory.

A system that uses inexact information as human judgment and evaluation can be modelled by utilizing the concept of a fuzzy graph. Then using the fuzzy graph model, it will be possible to quantitatively process fuzzy information. To clarify the global features of a fuzzy graph, we represent it as an approximate graph and extract the characteristics of fuzzy graph, such as relational and similar structures. Information processing requires rapid drawing and comprehensive display of fuzzy graphs and related graphs. A computer-aided analysis system could be effective, but such a system has not yet been developed.

To fill these needs, we propose a method for approximate analysis of a fuzzy graph and a computer-aided system that analyzes fuzzy graphs by means of human interaction. In our system using computer, it is easy to draw a partition tree for a cluster analysis and to draw an approximate fuzzy graph. An approximate fuzzy graph is firstly represented as a shape of its graph which all the nodes are arranged on a circumference. This shape can also be transformed through human interaction into one in which all the nodes are rearranged on the intersection of the lattices.

This paper describes a method for analyzing fuzzy graphs in section 2, analysis system in section 3, method of human interaction methodology in section 4 and a concluding remark in section 5.

2. METHOD FOR ANALYZING FUZZY GRAPHS

A system with a relation of inexact information among elements, for example, a system including the human relations of a specified group, can be represented as a fuzzy graph and be easily clarified through such a global feature as a relational or similar structure.

2.1 Fuzzy Graph

A graph is defined as a shape drawn to include some points and the lines between them. Usually, a point is called a node and a line is an arc. A graph is defined mathematically as follows: given a set V of nodes and a mapping relation M between elements v_i of V, a graph is defined as G=(V, M). $M(v_i)$ defines the mapping relations from an element v_i of a set V to other elements.

A graph is called a crisp graph if all the values of arcs is only 1 or 0. A graph is called a fuzzy graph if its value is between 0 and 1.

2.2 Analysis Method

In this section, we would illustrate an analysis by methods of partition tree and approximate expression for a fuzzy graph.

2.2.1 Partition tree

A value of elements f_{ij} of transitive closure matrix $\hat{F}$ means maximum one of relations between arc x_i and arcy_i. We defines c-cut matrix F^c as follows:

$$F^c = (f_{ij}),$$

$$f_{ij}^{C} = \begin{cases} 1 & (f_{ij} > c^*) \\ 0 & (f_{ij} < c^*) \end{cases}$$

The relation Rc of the c-cut matrix F^c for the transitive closure matrix $\hat{F}$ becomes a similarity relation. The fuzzy quotient set X/Rc represents the relation of the cluster that classifies elements with a similarity of more than c. An example of a partition tree for a fuzzy matrix F is shown in Fig. 1.

$$\begin{array}{cc} & \begin{array}{ccc} p & q & r \end{array} \\ \hat{F} = & \begin{bmatrix} 1 & 0.6 & 0.6 \\ 0.6 & 1 & 0.8 \\ 0.6 & 0.8 & 1 \end{bmatrix} \begin{array}{c} p \\ q \\ r \end{array} \end{array}$$

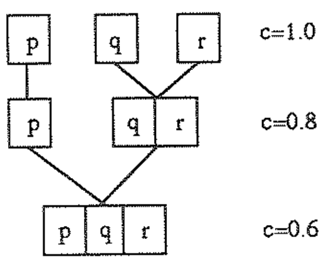

Figure 1. Example of partition tree

2.2.2 Approximate expression for a fuzzy graph

In order to analyze the global structure of the fuzzy graph F, we firstly define the p-graph F^P of F for $p \in [0, 0.5)$.

$$f_{ij} = \begin{cases} 1 & (f_{ij} > 0.5 + p^*) \\ 0 & (f_{ij} < 0.5 - p^*) \\ 0.5 & (\text{otherwise}) \end{cases}$$

We also define two functions. One is the distance function d(p) between F and F^P ;

$$d(p) = 2\sum \left| f_{ij}^P - f_{ij} \right| / (n^2 - n)$$

and another is the vagueness function e(p) of F^P ;

$$e(p) = v(p) / (n^2 - n)$$

$$\text{where } v(p) = |V| \text{ and } V = \left\{ f_{ij}^P \in F^P \;;\; f_{ij}^P \neq 0, 1 \right\}.$$

Here, we have the following properties concerning d(p) and e(p).

$$d_m = d(0.25) \leqq d(p), \quad \forall p \in [0, 0.5]$$

$$p < q \Rightarrow e(p) \leqq e(q), \quad \forall p, \quad q \in [0, 0.5]$$

Secondly, we define the membership function of the similitude degree between F and F^P;

$$f_D(p) = \{d_M - d(p)\} / \{d_M - d_m\} \in [0, 1]$$

and also define the membership function of the crispness degree of F^P;

$$f_E(p) = \{e_M - e(p)\} / \{e_M - e_m\} \in [0, 1]$$

$$\text{where } d_M = \mathrm{Max}\{d(0), d(0.5)\}, \; d_m = d(0.25),$$

$$e_M = e(0.5) \text{ and } e_m = e(0).$$

Thence, the maximal decision p* of the fuzzy decision $f_G(p) = f_D(p) \wedge f_E(p)$ should be an optimal value of p, and we could optimally have the approximate ternary graph F* of F by using p*.

$$f_{ij}^* = \begin{cases} 1 & (f_{ij} > 0.5 + p^*) \\ 0 & (f_{ij} < 0.5 - p^*) \\ 0.5 & (\text{otherwise}) \end{cases}$$

2.3 Example of Analysis for Fuzzy Gaph

In this section, we will discuss the approximate expression for a fuzzy graph F shown in Fig. 2 whose fuzzy matrix F is listed in Table 1.

Table 1. Fuzzy Matrix F

	1	2	3	4
1	*	0.86	1.00	1.00
2	0.48	*	0.53	0.89
3	0.67	0.76	*	0.94
4	0.34	0.59	0.53	*

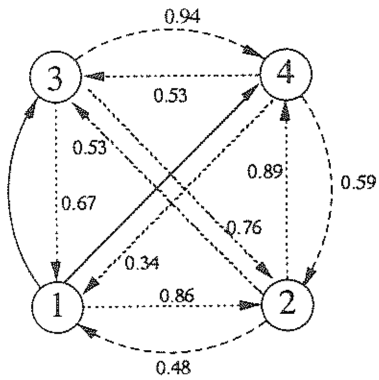

Figure 2. Fuzzy Graph F

Based on the method mentioned above, we can make the similitude function d(p) between F and its p-graph F^p and the vagueness function e(p) of F.

According to the functions d(p) and e(p), we can have their membership functions $f_D(p)$ and $f_E(p)$, and the fuzzy decision $f_G(p) = f_D(p) \wedge f_E(p)$.

By the maximal decision of the fuzzy decision $f_G(p)$, we can obtain the optimal value p* = 0.03 of p and we can have the approximate ternary graph F* shown in Fig.3.

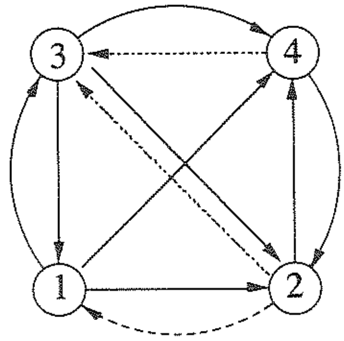

Figure 3. Fuzzy Graph F*

In this way, we can effectively analyze the main structure of a fuzzy graph F and construct its ternary graph F*, and it should be a reasonably approximate expression of a fuzzy graph F.

3. ANALYSIS SYSTEM

Our goal is to develop a general purpose, computer-aided analysis system for analyzing fuzzy graph through human interaction, which can efficiently analyze its features. The system that we have developed to date can process only similar-structure and related-structure analyses.

3.1 Functions of the System

Fuzzy graphs have been traditionally analyzed by the trial-and-error method, because an analysis algorithm has not yet been established. Hence we are developing a system to help easy and comprehensive analyses for a fuzzy graph through human interaction. Our system has the following functions:

(a) Input of a fuzzy matrix
(b) Correction and change of an input fuzzy matrix
(c) Drawing and display of a partition tree(for similar-structure analysis)
(d) Drawing and display of an approximated fuzzy graph (for related-structure analysis)
(e) Changing the shape of an approximated fuzzy graph into another shape through human interaction.

3.2 Architecture of the System

The system is composed of three parts: input, process, and output. Human interaction are carried out through the graphic user interface.

(1) Input: A fuzzy matrix is input by keyboard. If the input data is in error, an user can correct it by keyboard and mouse. The system executes either similar-structure or related-structure analysis according to an user direction.

(2) processing: The system executes either similar-structure or related-structure analysis.

The algorithm of similar-structure analysis processing is as follows:

(a) Compute a transitive closure matrix $\hat{F}$

(b) Determine the degree of the rank Rz of the cluster analysis

(c) Determine the order of the node in the first rank

(d) Cluster the nodes according to previously determined the Rz

(e) Classify the Rz and draw and display a partition tree

The algorithm of related-structure analysis is as follows:

(a) Compute the p-graph F^p for the fuzzy graph F

(b) Compute the distance function d(p) between F and F^p

(c) Compute the vagueness function e(p) of F

(d) Compute the maximal decision p* based on d(p) and e(p)

(e) Compute an approximate N-valued (ternary) graph and draw and display it

(f) Freely change the shape of the graph for easy analysis

(3) Human interaction: A partition tree is firstly displayed in similar-structure analysis as shown in Fig. 1 and Fig.5, and a fundamental fuzzy graph is first displayed in related-structure analysis as shown in Fig.5. These shapes are rearranged and/or changed in order to easily grasp the global structure of the fuzzy graph and to comprehensively understand it.

4. METHD OF HUMAN INTERACTION

The system's functions and performance are described in the following sections. For a fuzzy graph on a workstation, the maximum permissible number of nodes is about 50 nodes. A fuzzy graph with 50 nodes is shown in Fig.4. The structure of fuzzy graph with many nodes and arcs is very difficult to understand.

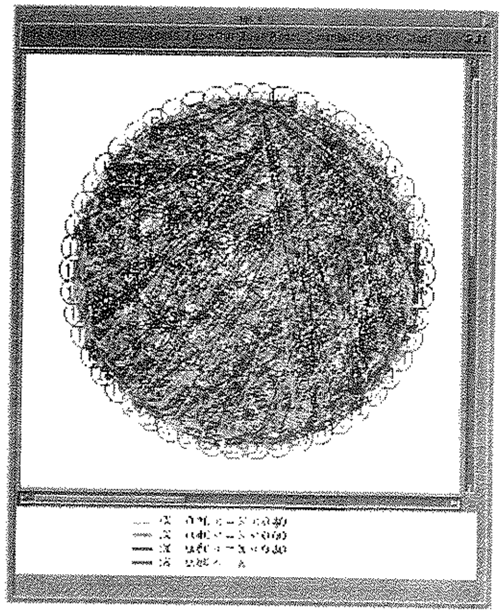

Figure 4. Fuzzy Graph with 50 Nodes

4.1 Correction of Input Data

An original fuzzy graph is drawn directly on the display device based on the input data from a fuzzy matrix. Whether the input data is in error or not can be perceive immediately from the shape of the fuzzy graph. The fuzzy graph can be changed by using the fuzzy matrix.

4.2 Fundamental Functions

(1) Color display of arc: It is very difficult to display on one device simultaneously all the approximated values of the arcs of a fuzzy graph. The approximated values are divided into N classes. Each arc is displayed by its own color so that they can be immediately distinguished. This also helps us easily and comprehensively understand the global structure of the fuzzy graph.

(2) Zoom in and zoom out functions: When the shape of an approximated fuzzy graph, with its many nodes and arcs, is drawn on a display device, its structure is difficult to understand because it is small and complex. So, the system can zoom in and zoom out the fuzzy graph on a display device. We can then look at easily a fuzzy graph through a suitable shape, as transformed by enlarging or reducing it.

(3) Display of partition tree: This system can compute the degree of similarities between the nodes and draw a partition tree on a display device based on input data. It can also make a cluster of fuzzy graphs depending upon the degree of similarities, and display its shape. Thus we can

easily perceive the degree of similarities of a fuzzy graph at a glance and analyze its global structure (Fig.5).

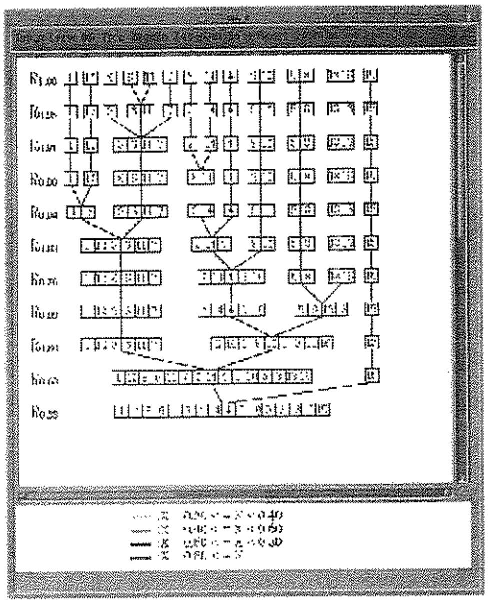

Figure 5. Partition Tree

(4) Display of c-cut graph: A fuzzy graph is approximated by a graph having arcs of N-classifications. The system can automatically eliminate arcs whose values are smaller than a specified value (threshold value) and display the resulting fuzzy graph. This feature also enhances the analysis of the global structure of the fuzzy graph (Fig. 6).

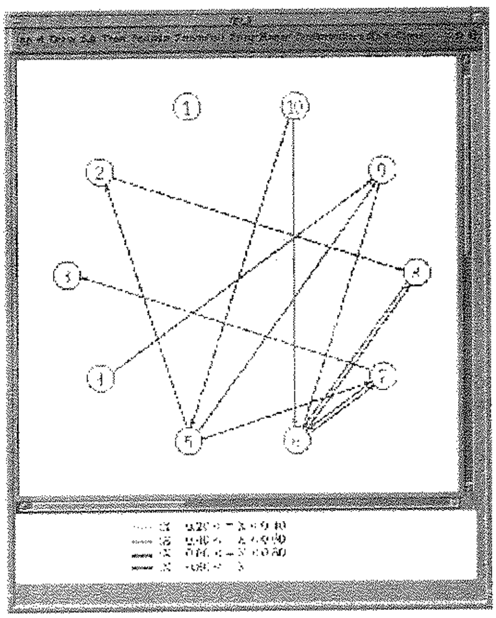

Figure 6. C-cut Graph

4.3 Functions for Transforming a Fuzzy Graph

(1) Display based on a specified node: This system can display only a specified node and the arcs connected to it. This function is efficient in analyzing a fuzzy graph in teams of the specified node (Fig.7).

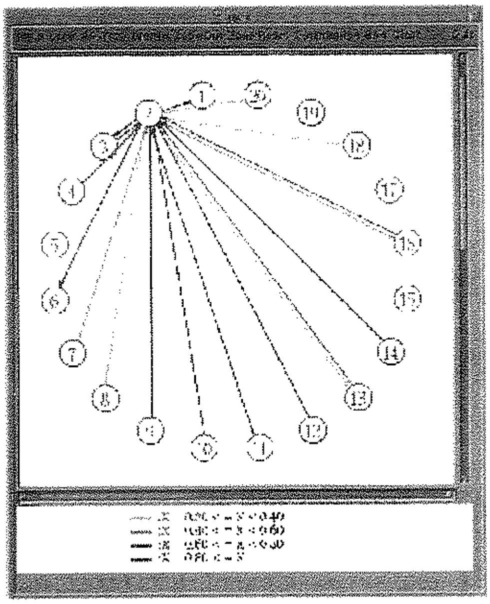

Figure 7. Specified Node Dsplay

(2) Rearrangement of nodes for reducing cross-arcs: It is generally desirable to reduce the numbers of cross-arcs in order to easily grasp the global structure of fuzzy graph. In this system, computer counts the number of cross-arcs in a fuzzy graph and automatically rearranges the nodes so as to reduce the cross-arcs as much as possible. We can more easily understand the structure of the fuzzy graph.

(3) Rearrangement of nodes by hand: Shapes of fuzzy graph on display are desired to

rearrange and/or change in order to easily grasp global structure of fuzzy graph and to comprehensively understand it through human interaction. This system can rearrange nodes according to the movement of a mouse by hand, allowing us to easily look at any type of shape.

(4) Making and breaking clusters for a fuzzy graph: Clusters of a fuzzy graph can be made and/or broken by specifying the elements of a fuzzy matrix on the multi-windows of the display by hand (Fig. 8). In this Figure the encircled numbers denote the usual nodes of a fuzzy graph and the numbers enclosed by squares denote clusters. We can easily look at various kinds of shape for a fuzzy graph, for example a shape composed of only nodes and arcs, or shape composed of nodes and clusters.

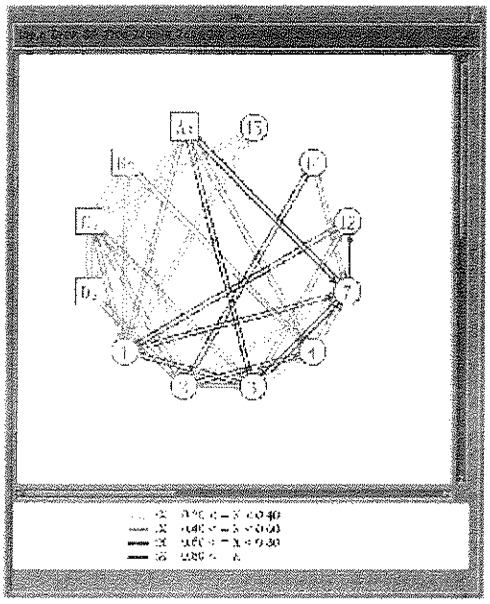

Figure 8. Clusters for a Fuzzy Graph

(5) Arrangement of nodes on the lattice intersections: Experience tells us that the shape of a graph, which all the node is arranged on the intersection of the lattices, is relatively easy to analyze. This system can transform the shape of a fuzzy graph through human interaction. All nodes positions must be specified by hand (Fig. 9).

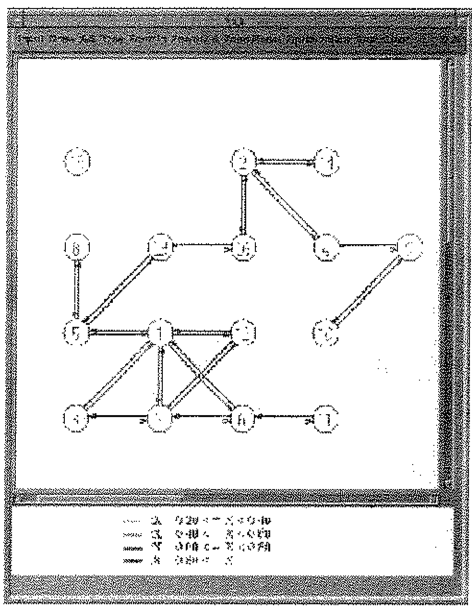

Figure 9. Nodes Arrangemen on the Lattices

5. CONCLUDING REMARKS

We propose a method of approximate analysis for a fuzzy graph. This method can effectively analyze fuzzy information and make a reasonable decision on fuzzy phenomena by applying the approximate graph F*. We have also developed a computer-aided analysis system for analyzing a fuzzy graph through human interaction. Using this system, it is possible to analyze speedily, comprehensively and easily similar-structure and related-structure of fuzzy graphs.

Two main problems remain to be solved in the future, to enhance this analysis system. One is the improvement of the human interface in the rearranging nodes to reduce the numbers of the cross-arcs in a fuzzy graph with many nodes, and in the arranging nodes on the intersection of the lattice. The other is addition of other functions for our purpose.

6. ACKNOLEGDEMENTS

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